Design Manual to EC2
BS EN 1992-1-1:2004

Version 3.1
January 2018
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The LinkStudPSR System offers customers a fast, easy and extremely cost effective method of providing Punching Shear Reinforcement around columns and piles within flat slabs and post-tensioned slabs, at slab to shearwall junctions, beam to column junctions and within footings and foundation slabs.

The LinkStudPSR System comprises short lengths of carbon steel deformed bar reinforcement with end anchorages provided by enlarged, hot forged heads at both ends, giving a cross-sectional area ratio of 9:1. These stud heads anchor securely in the slab, eliminating slippage and providing greater resistance to punching shear.

The double-headed LinkStudPSR shear studs are welded to carrier / spacer rails to allow them to be located correctly and to be supported by the top flexural reinforcement.

LinkStudPSR is a technologically advanced and proven system - a fully-tested, fully-accredited, fully-traceable Punching Shear Reinforcement System approved by CARES for use in reinforced concrete slabs designed in accordance with EC2 design standards.

Through our total focus on Punching Shear Reinforcement we have become experts in our field, with unparalleled experience in the design of PSR schemes and a thorough knowledge of the intricacies and complexities of the EC2 design standards. We are pleased to be able to offer you this expertise as a cornerstone of the LinkStudPSR package.

From application advice and design guidance, through proposal drawings, calculations and quotations, to working drawings and site support, you can depend on LinkStudPSR for all your Punching Shear Reinforcement needs.

Kind Regards

Dariusz Nowik MSc (Eng)
Senior Design Engineer
LinkStudPSR Limited
Greek Symbols

- $\alpha$ - angle between rails in quarter
- $\alpha_{cc}$ - coefficient for long term effects
- $\beta$ - eccentricity factor
- $\gamma_c$ - partial factor for material for ULS
- $\delta$ - when a hole is present $\delta$ indicates an angle between the tangent lines of the ‘dead zone’
- $\sigma_{cp}$ - the normal concrete stresses in the critical section
- $\sigma_{cx}, \sigma_{cy}$ - the normal concrete stresses in the critical section in x- or y- directions
- $\rho_l$ - tension reinforcement ratio
- $\rho_{lx}, \rho_{ly}$ - ratio of tension reinforcement in both directions

Latin Symbols

- $A_c$ - area of concrete according to the definition of $N_{ed}$
- $a_{lx}, a_{ly}$ - distance to slab edge in x- and y- direction
- $A_{sw}$ - area of one perimeter of shear reinforcement around the loaded area
- $A_{sw min}$ - minimum area of one perimeter of shear reinforcement around the loaded area
- $A_{sw1}$ - area of one shear stud
- $A_{sw1 min}$ - minimum area of one shear stud
- $A_{sx}, A_{sy}$ - area of T1 and T2 main reinforcement per width of the loaded area + 3d each face
- $b$ - considered width of the slab
- $B$ - effective part of the perimeter of the square or rectangular loaded area facing:
  - $B_N$ - north, $B_E$ - east, $B_S$ - south, $B_W$ - west.
- $B_C$ - effective part of the perimeter of the circular loaded area facing:
  - $B_{CN}$ - north, $B_{CE}$ - east, $B_{CS}$ - south, $B_{CW}$ - west.
- $c$ - diameter of the circular column
- $c_1, c_2$ - square/ rectangular column dimensions
- $C_{Rd.c}$ - NA to BS EN 1992-1-1-2004 6.4.4 (1)
- $d$ - effective depth of the slab
- $d_x, d_y$ - effective depths of the reinforcement in two orthogonal directions
- $f_{cd}$ - design compressive strength of concrete
- $f_{ck}$ - characteristic compressive cylinder strength of concrete at 28 days (BS EN 1992-1-1-2004, table 3.1)
- $f_{yk}$ - characteristic tensile strength of the reinforcement
- $f_{wd}$ - design strength of the punching reinforcement
- $f_{wd, ef}$ - effective design strength of the punching reinforcement
- $h$ - slab depth
- $k, k_1$ - factors 6 .4.4 (1)
- $k_2$ - coefficient dependent on the ratio between the column dimensions $c_1$ and $c_2$
- $l_1, l_2$ - hole dimensions
- $M_{ed}$ - bending moment
- $n$ - number of rails with the first stud at a maximum of 0.5 d from the loaded area face
- $N_{ed,x}, N_{ed,y}$ - longitudinal forces across the full bay for internal columns and the longitudinal force across the control section for edge columns
- $n_s$ - number of segments around $u_{out}$ on a circular layout pattern
- $n_t$ - number of studs per rail
p - distance from the centre point of a circular layout pattern to $u_{out}$ perimeter
$p_1$ - distance from the loaded area face to the last stud on the rail
$p_2$ - distance from the loaded area face to the centre point of a circular layout pattern
$r_3$ - distance to the 3rd stud
$r_{last}$ - distance to the last stud
$s$ - length of equal segments around $u_{out}$ on a circular layout pattern
$s_3$ - maximum tangential spacing inside the basic control perimeter (usually between 3rd studs)
$s_{last}$ - maximum tangential spacing between last studs outside the basic control perimeter
$s_t$ - tangential studs spacing
$s_r$ - radial studs spacing
$t_{tc}, t_{bc}$ - top cover, bottom cover
$u_0$ - loaded area perimeter
$u_{0\,red}$ - part of the loaded area perimeter within the "dead zone"
$u_1$ - basic control perimeter
$u_{1\,red}$ - part of the basic control perimeter within the "dead zone"
$u_{out}$ - control perimeter beyond which shear reinforcement is not required
$u_{out2}$ - extended $u_{out}$ control perimeter which takes account of the presence of a hole
$u_{out,ef}$ - $u_{out}$ for cruciform pattern when the distance between the last studs is greater than 2d
$V_{Ed}$ - design value of the shear force
$V_{Ed\,0}$ - design value of the shear stress at the loaded area face
$V_{Ed\,1}$ - design value of the shear stress at the basic control perimeter
$V_{min}$ - minimum concrete shear stress resistance
$V_{Rd\,c}$ - design value of the punching shear resistance of a slab without shear reinforcement at the basic control perimeter
$V_{Rd\,cs}$ - design value of the punching shear resistance of a slab with shear reinforcement at the basic control perimeter
$V_{Rd\,max}$ - design value of the maximum punching shear resistance at the loaded area face
$W_1$ - corresponds to a plastic distribution of the shear stress as described in BS EN-1992-1-1-2004, Fig. 6.19.
The following information is required to design shear reinforcement:

- $V_{Ed}$ - design value of the shear load
- the shape and size of the loaded area
- $f_{ck}$ - the characteristic compressive cylinder strength of the concrete
- the tension reinforcement diameter and spacing in both directions within 3d from the loaded area face
- the thickness of the slab, top and bottom cover to the main reinforcement
- the distance to the slab edge in both directions
- the location and size of any hole(s) within 6d from the loaded area face
- the location of any step in the slab or any changes to the slab thickness

We assume that:

- the thickness of the slab is equal or greater than 200mm
- the loads given have been factored with EC2 factors
- the loads given do not include the loads from the column above
- the concrete slab has not been made using lightweight aggregate
- the main reinforcement bars are placed accordingly to the detailing rules described in 9.4.1 and 9.4.2 (1)

In order to design using the proper value of the shear stress, we recommend engineers provide us with values and directions of the bending moments or the value of eccentricity factor $\beta$ as calculated by the Project Engineer – if these values are not provided the approximate value of $\beta$ will be used.
For structures where lateral stability does not depend on frame action between the slabs and the columns, and where the adjacent spans do not differ in length by more than 25%, approximate values for $\beta$ may be used.

The recommended values of the eccentricity factor $\beta$ are listed below:

(6.4.3 (6))

Internal column $\beta = 1.15$

Edge column $\beta = 1.4$

External corner column $\beta = 1.5$

Internal corner column $\beta = 1.275$ (the value found by interpolation)

a) Beta factor for rectangular and square columns and cantilever slab

If the slab edge does not line up with the loaded area face, the following rules may apply:

**Edge conditions**

If $a_{slx} \geq (c_2 + 2nd)/2$ than $\beta = 1.15$, If $a_{sly} = 0$ than $\beta = 1.4$.

Beta factor for $a_{slx}$ between the above values may be found by interpolation.

**External corner conditions**

$\beta = \max (\beta_x, \beta_y, \beta_{xy})$

Where:

$\beta_x = 1.4 - 0.25 \times (2 \times a_{slx} / (c_2 + 2nd))$

$\beta_y = 1.4 - 0.25 \times (2 \times a_{sly} / (c_1 + 2nd))$

$\beta_{xy} = 1.5 - 0.35 \times (a_{slx} + a_{sly}) / (c_1 + c_2 + 3nd)$

**Internal corner conditions**

If: $a_{slx} \leq 0$, and $a_{sly} \leq 0$ than $\beta = 1.275$

If: $a_{slx} \leq 0$, and $a_{sly} \leq c_1 - 2d$, or $a_{sly} \leq 0$, and $a_{slx} \leq c_1 - 2d$, than $\beta = 1.4$

Beta factor for $a_{slx}, a_{sly}$ between the above values may be found by interpolation.
b) Beta factor for circular columns and cantilever slab

Similar rules apply for the circular loaded area. See the details below:

**Edge conditions**

If $a_{sdx} \geq \pi/4(c + 4d)$ than $\beta = 1.15$, if $a_{sdx} \leq c/2$ than $\beta = 1.4$.

Beta factor for $a_{sdx}$ between the above values may be found by interpolation.

**External corner conditions**

$\beta = \max (\beta_x, \beta_y, \beta_{xy})$

Where:

$\beta_x = 1.4 - 0.25 \cdot 4 \cdot a_{sdx} / (\pi(c + 4d))$

$\beta_y = 1.4 - 0.25 \cdot 4 \cdot a_{sdy} / (\pi(c + 4d))$

$\beta_{xy} = 1.5 - 0.35 \cdot (a_{sdx} + a_{sdy}) / (3/4 \cdot \pi(c + 4d))$

**Internal corner conditions**

If: $a_{sdx} \geq \pi/4(c + 4d)$, or $a_{sdy} \geq \pi/4(c + 4d)$, or $a_{sdx} + a_{sdy} \geq \pi/4(c + 4d)$, and $a_{sdx} > 0$ and $a_{sdy} > 0$ than $\beta = 1.15$

If: $a_{sdx} \leq c/2$, and $a_{sdx} \geq -c/2$, and $a_{sdy} \leq c/2$, and $a_{sdy} \geq -c/2$ then $\beta = 1.275$

If: $a_{sdx} \leq c/2$, and $a_{sdx} \geq -c/2$, and $a_{sdy} \leq -2d - c/2$, or $a_{sdy} \leq c/2$, and $a_{sdy} \geq -c/2$, and $a_{sdx} \leq -2d - c/2$, than $\beta = 1.4$

Beta factor for $a_{sdx}$, $a_{sdy}$ between the above values may be found by interpolation.
c) Calculating eccentricity factor $\beta$ with bending moment present

The Eccentricity Factor $\beta$ may be calculated by taking existing bending moments into account. The expressions below give a more accurate value of $\beta$ factor.

**Internal conditions**

The Eccentricity Factor should be calculated as follows:

$$\beta = 1 + k_2^2 \frac{M_{Ed} u_1}{(V_{Ed} W_1)}$$

6.4.3 equation 6.39

Where:

- $M_{Ed}$ - bending moment
- $V_{Ed}$ - shear force
- $W_1$ - corresponds to a shear distribution (sum of multiplication of basic control perimeter lengths and the distance from gravity centre to the axis about which the moments act).
- $u_1$ - basic control perimeter
- $k_2$ - coefficient dependent on the ratio between the column dim. $c_1$ and $c_2$ (see tab.6.1)

| $c_1/c_2$ | 0.5 | 1.0 | 2.0 | $\geq 3.0$
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</thead>
<tbody>
<tr>
<td>$k_2$</td>
<td>0.45</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

$W_1$ for rectangular loaded area is equal:

$$W_1 = 0.5c_1^2 + c_1c_2 + 4c_2d + 16d^2 + 2\pi d c_1$$

6.4.3 equation 6.41

$W_1$ for circular loaded area is equal:

$$W_1 = (c + 4d)^2$$

Therefore the Eccentricity Factor for circular loaded area is equal:

$$\beta = 1 + 0.6\pi \frac{M_{Ed}}{(V_{Ed}(c + 4d))}$$

6.4.3 equation 6.42

In cases where moments in both directions for rectangular loaded area are present, the Eccentricity Factor should be determined using the expression:

$$\beta = 1 + 1.8\sqrt{\left(\frac{M_{Edx}}{(V_{Ed}(c_1+4d))^2} + \frac{M_{Edy}}{(V_{Ed}(c_2+4d))^2}\right)}$$

6.4.3 equation 6.43

Where:

- $M_{Edx}$ - bending moment about $x$ axis (parallel to $c_1$)
- $M_{Edy}$ - bending moment about $y$ axis (parallel to $c_2$

For a circular loaded area $c_1 = c_2$

**Edge conditions**

Where the eccentricity in both orthogonal directions is present, the Eccentricity Factor should be determined using the expression:

$$\beta = u_1/u_1^* + k_2^* e_{par} u_1 \sqrt{N_1}$$

6.4.3 equation 6.44

Where:

- $u_1$ - reduced basic control perimeter (drawing 6.20a)
- $e_{par}$ - eccentricity from the moment perpendicular to the slab edge $e_{par} = M_{Ed}/V_{Ed}$
- $k_2^*$ - may be determined from tab. 6.1 with the ratio $c_1/c_2$ replaced by $c_1/2c_2$
$W_1$ - is calculated for the basic control perimeter about the axis perpendicular to the slab edge; $W_1$ for rectangular loaded area is equal:

$$W_1 = 0.25c_2^2 + c_1c_2 + 4c_1d + 8d^2 + \pi d^2$$

$W_1$ for circular loaded area is equal:

$$W_1 = c^2 + 6d c + 8d^2$$

6.4.3 equation 6.45

If the bending moment about the axis parallel to the slab edge exists, eccentricity is toward the interior and there is no other bending moment, then the punching force is uniform along the reduced control perimeter $u_1$ (see drawing 6.20a). Factor $\beta$ is equal:

$$\beta = 1 + k_2 \frac{M_{Ed}}{u_1} \frac{V_{Ed}}{W_1}$$

Where:
- $k_2$ - may be determined from tab. 6.1 with the ratio $c_1 / c_2$ replaced by $c_1 / 2c_2$
- $W_1$ - is calculated for the reduced control perimeter about the axis parallel to the slab edge located in the centroid of the reduced control perimeter.

**Corner conditions**

In corner conditions, when eccentricity is towards the interior of the slab, the Eccentricity Factor may be considered as:

$$\beta = \frac{u_1}{u_1^*}$$

6.4.3 equation 6.39

If the eccentricity is towards the exterior, expression 6.39 applies.

$$\beta = 1 + k_2 \frac{M_{Ed}}{u_1} \frac{V_{Ed}}{W_1}$$

$W_1$ for a rectangular loaded area (external corner conditions) is equal:

$$W_1 = 0.5c_1c_2 + 2d c_1 + 0.25c_2^2 + 4d^2 + 0.5d c_2$$

$W_1$ for a circular loaded area (external corner conditions) is equal:

$$W_1 = 11c^2/8 + 9d c + 16d^2$$
The effective depth of the slab is assumed to be constant and may be taken as:
\[ d = \frac{(d_x + d_y)}{2} \]
where:
\[ d_x = h - t_{tc} - T1/2 \]
\[ d_y = h - t_{tc} - T1 - T2/2 \]
\[ t_{tc} - top cover \]
a) Perimeter of the Loaded Area

Conditions for rectangular / square columns

<table>
<thead>
<tr>
<th>Condition</th>
<th>Internal</th>
<th>Edge</th>
<th>External corner</th>
<th>Internal corner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>$u_0 = 2c_1 + 2c_2$</td>
<td>$u_0 = B_N + B_S + c_2$</td>
<td>$u_0 = B_E + B_S$</td>
<td>$u_0 = c_1 + c_2 + B_N + B_W$</td>
</tr>
</tbody>
</table>

Where:

- $B_N, B_S = \min. (c_1, c_1 + a_{slx}, 1.5 \, d)$
- $B_E, B_W = \min. (c_2, c_2 + a_{sly}, 1.5 \, d)$

Conditions for circular columns

<table>
<thead>
<tr>
<th>Condition</th>
<th>Internal</th>
<th>Edge</th>
<th>External corner</th>
<th>Internal corner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>$u_0 = \pi c$</td>
<td>$u_0 = \pi c/4 + B_N + B_S$</td>
<td>$u_0 = B_E + B_S$</td>
<td>$u_0 = \pi c/2 + B_N + B_W$</td>
</tr>
</tbody>
</table>

Where:

- $B_N, B_S = \min. (0.25 \pi \, c, 0.125 \pi \, c + a_{slx}, 1.5 \, d)$
- $B_E, B_W = \min. (0.25 \pi \, c, 0.125 \pi \, c + a_{sly}, 1.5 \, d)$

Note: if any hole within 6d from the face of the loaded area is present, the loaded area perimeter should be reduced (see 6.4.2 (3) EC2 and section 13)

Note: if the slab edge is offset at least 2d from the loaded area face then its presence should be ignored when calculating $u_0$.

b) Design value of the shear stress at the Loaded Area

$$v_{Ed,0} = \beta \frac{V_{Ed}}{u_0 d}$$

6.4.5 (3) 6.53
c) Design value of the maximum punching shear resistance at the Loaded Area

\[ f_{cd} = \alpha_{cc} f_{ck} / \gamma_c \]

\[ \gamma_c = 1.5 \]

\[ \alpha_{cc} = 1 \]

\[ v_{Rd,max} = 0.3 f_{cd} (1 - (f_{ck} / 250)) \]

NA to BS EN 1992-1-1-2004 6.4.5 (3) note, 6.2.2(6) 6.6N

Where:

- \( f_{cd} \) – the value of the design compressive strength of concrete. 3.1.6
- \( f_{ck} \) - the characteristic compressive cylinder strength of concrete at 28 days. table 3.1
- \( \alpha_{cc} \) – the coefficient for long term effects NA to BS EN 1992-1-1-2004 3.1.6 (1)P
- \( \gamma_c \) – the partial factor for material for ULS 2.4.2.4 table 2.1N

Please note that \( f_{cd} \) is limited to the strength of C50/60, unless otherwise proven.

If \( v_{Ed,0} > v_{Rd,max} \) - the slab depth or the column size must be increased. 6.4.3 (2a)
a) Basic Control Perimeter length

The Basic Control Perimeter is located 2d from the loaded area.

**Internal column**

\[ u_1 = 2c_1 + 2c_2 + 4\pi d \]

**Edge column**

\[ u_1 = \pi \cdot (4d + c) \]

\[ u_1 = 2c_1 + c_2 + 2\pi d + 2a_{\text{slx}} \]

\[ u_1 = \pi /2 \cdot (4d + c) + 2a_{\text{slx}} \]
7. Punching Shear at the Basic Control Perimeter without Reinforcement

External corner column

\[ u_1 = c_1 + c_2 + \pi d + a_{slx} + a_{aly} \]

\[ u_1 = \frac{\pi}{4} \cdot (4d + c) + a_{slx} + a_{aly} \]

Internal corner column

\[ u_1 = 2(c_1 + c_2) + 3\pi d + a_{slx} + a_{aly} \]

\[ u_1 = \frac{3}{4}\pi \cdot (4d + c) + a_{slx} + a_{aly} \]

Note: if any hole within 6d from the face of the loaded area is present, the loaded area perimeter should be reduced (see 6.4.2 (3) EC2 and section 13)
b) Design value of the maximum shear stress at the Basic Control Perimeter

\[ \nu_{Ed,1} = \beta \frac{V_{Ed}}{u_1 d} \]  
6.4.3 (3), (6.38)

c) Punching Shear Resistance at the Basic Control Perimeter

\[ k = 1 + \sqrt{200/d} \leq 2 \]
\[ \nu_{min} = 0.035k^{3/2} f_{ck}^{1/2} \]  
6.2.2 (1), (6.3N)
\[ C_{Rd,c} = 0.18/\gamma_c \]
\[ k_1 = 0.1 \]
\[ \sigma_{cx} = N_{ed,x}/A_{cx} \]
\[ \sigma_{cy} = N_{ed,y}/A_{cy} \]
\[ \sigma_{cp} = (\sigma_{cx} + \sigma_{cy})/2 \]
\[ \rho_x = A_{sx}/(bd_x) \]
\[ \rho_y = A_{sy}/(bd_y) \]
\[ \rho_i = \sqrt{(\rho_x \rho_y)} \leq 0.02 \]
\[ \nu_{Rd,c} = C_{Rd,c} \left( 100 \rho_i f_{ck}^{1/3} + k_1 \sigma_{cp} \right) \geq (\nu_{min} + k_1 \sigma_{cp}) \]  
6.4.4 (1), (6.47)

Where:
\( \sigma_{cx}, \sigma_{cy} \) - the normal concrete stress in the critical section in x- and y- directions (MPa, positive if compression)
\( N_{ed,x}, N_{ed,y} \) - the longitudinal forces across the full bay for internal columns and the longitudinal force across the control section for edge columns. The force may be from a load or pre-stressing action.
\( A_c \) - the area of concrete according to the definition of \( N_{ed} \)
\( \rho_x, \rho_y \) - the ratio of tension reinforcement in both directions
\( b \) - the width of the loaded area + 3d each side
\( \rho_i \) - the tension reinforcement ratio
If \( \nu_{Ed,1} < \nu_{Rd,c} \) - punching shear reinforcement is not required  
6.4.3 (2b)
a) Design value of the punching shear resistance at the Basic Control Perimeter

\[ f_{yw,\text{ef}} = 250 + 0.25 \, d \leq f_{yw} = \frac{f_y}{1.15} \]

\[ \nu_{Rd,cs} = 0.75 \, \nu_{Rd,c} + 1.5 \, (d / s_i) \, A_{sw} \, f_{yw,\text{ef}} \, (1 / (u_1 \, d)) \geq \nu_{Ed,1} \]

Where:
- \( s_r \): radial studs spacing
- \( f_{yw,\text{ef}} \): the effective design strength of the punching reinforcement.
- \( A_{sw} \): area of one perimeter of the shear reinforcement around the loaded area.

b) The area of the shear reinforcement

\[ A_{sw1,\text{min}} = 0.08 \, (s_t \, s_i) \, \sqrt{f_{ck}} / 1.5 \, f_y \]

\[ A_{sw1} = (\nu_{Ed,1} - 0.75 \, \nu_{Rd,c}) \, u_1 \, s_r / (1.5 \, f_{yw,\text{ef}} \, n) \leq A_{sw1,\text{min}} \]

Where:
- \( s_t \): tangential studs spacing
- \( n \): number of rails with the first stud at a maximum of 0.5 \( d \) from the loaded area face.
- \( A_{sw1} \): area of one shear stud.
- \( A_{sw1,\text{min}} \): the minimum area of one shear stud.
- \( f_y \): the characteristic tensile strength of reinforcement.
Shear reinforcement should be detailed in accordance with BS EN 1992-1-1:2004 6.4.5 (4), 9.4.3, and NA to BS EN 1992-1-1-2004 6.4.5 (4).

The first stud should be placed between 0.3d and 0.5d from the loaded area face. For cruciform pattern the recommended distance would be 0.35d.

There should be a minimum of two perimeters of reinforcement.

The radial spacing of the shear reinforcement (s_r) should not exceed 0.75d.

The tangential spacing of the shear reinforcement (s_t) should not exceed 1.5d within the Basic Control Perimeter.

The tangential spacing of the circular pattern shear reinforcement outside the Basic Control Perimeter should not exceed 2d. In the case of cruciform pattern, tangential spacing can go beyond 2d but this will affect $u_{out}$ by leaving gaps in the perimeter (see the drawing on page 20).

$u_{out}$ (or $u_{out,el}$) should be calculated using the following formula:

$$u_{out} = \beta \frac{V_{Ed}}{(V_{Rd,c} d)}$$

The outermost perimeter of shear reinforcement should be placed at a distance not greater than kd within $u_{out}$ (or $u_{out,el}$)

Where:

$k = 1.5$ unless the perimeter $u_{out}$ (or $u_{out,el}$) is located closer than 3d from the loaded area face. In this case the shear reinforcement should be placed in the zone 0.3d to 1.5d from the loaded area face.

The shape of the perimeter $u_{out}$ (or $u_{out,el}$) will depend on the arrangement of the shear reinforcement and on spacing limitations.
1.5d last shear stud to be located 1.5d from the basic control perimeter.

Additional reinforcement may be required to comply with spacing rules within basic control.

u₁ - basic control perimeter

u₂ - control perimeter

last shear stud to be located 1.5d from the u₂ control perimeter

additional reinforcement may be required to comply with spacing rules within basic control
a) Arranging studs

When $u_{out}$ is calculated, the perimeter can be drawn around the loaded area and the zone inside can be reinforced with shear studs. Care should be taken to ensure the shear reinforcement detailing rules described in section 9 are implemented.

The number of rails must be assumed in order to locate $u_{out}$.

The centre line of each corner rail must be in line with the pivot point located $p_2$ from the loaded area. $p_2$ is half of the shorter side of the loaded area side but not more than 0.75d.

The $u_{out}$ perimeter is created with segments of lines offset by 1.5d from the last perimeter of the shear studs. The length of these segments are not identical with the one in the middle of the quarter being the longest. This increase can be ignored on the basis that the larger perimeter will have an increased load capacity and is therefore considered to be a worst-case scenario.

The angle between the rail in quarter ($\alpha$) equates to:

$$\alpha = \frac{90}{n_s}$$

Where:

$n_s$ – the number of segments around $u_{out}$ on a circular layout pattern

For equations to calculate ‘s’ in different conditions, see Section 10 (b).

In order to locate the $u_{out}$ perimeter, ‘p’ must be calculated:

$$p = s / (2 \sin(\alpha/2))$$

Therefore ‘$p_1$’ – the distance from the loaded area face to last stud equates to:

$$p_1 = p - p_2 - 1.5d / (\cos(\alpha/2))$$

When $p_1$ is worked out, the number of studs ($n_t$) on the rail can easily be calculated using the following formula:

$$n_t > (p_1 - 0.5d) / 0.75d$$

The outcome should be rounded up to the nearest integer value.

The next step is to check the maximum tangential spacing between the studs on the rails in the two following cases

- inside basic control perimeter check if $s_3 \leq 1.5d$
- outside basic control perimeter check if $s_{last} \leq 2d$

Where:

$s_3$ – the maximum tangential spacing inside the basic control perimeter (usually between the 3rd studs)

$s_{last}$ – the maximum tangential spacing between the studs outside the basic control perimeter

To check tangential spacing, the distance from the pivot point to the last stud inside the basic control perimeter (usually the third) and the distance from the pivot point to the last stud outside the basic control perimeter must be calculated.

$$r_3 = p_2 / \cos(\alpha^*) + \text{distance to first stud} + 2 \cdot \text{studs spacing}$$

$$r_{last} = p_2 / \cos(\alpha^*) + \text{distance to first stud} + (\text{number of studs on rail} - 1) \cdot \text{studs spacing}$$

Where:

$r_3$ – the distance to the 3rd stud

$r_{last}$ – the distance to the last stud

$\alpha^*$ - angle might be multiplied with integer value depending on number of rails
The two rails (A and B) with the longest ‘r’ must be chosen (‘r’ for rails A and B are equal on the example below but this is not a rule). The spacing for both cases (inside and outside the basic control perimeter) should be calculated using the following formula:

\[ s_t = \sqrt{\left( r_A + r_B \cos(\alpha) \right)^2 + \left( r_B \sin(\alpha) \right)^2} \]

When the tangential spacing exceeds the maximum value, the number of rails should be increased or intermediate spacer rails should be used.
b) Calculating ‘s’ in different conditions

**Internal column**
\[ s = \frac{(u_{out} - 2(c_1+c_2) + 8p_2)}{n_s} \]

**Edge column**
\[ s = \frac{(u_{out} - 2c_1 - c_2 + 4p_2 - 2a_{slx})}{n_s} \]

**External corner column**
\[ s = \frac{(u_{out} - c_1 - c_2 + 2p_2 - a_{slx} - a_{sly})}{n_s} \]

**Internal corner column**
\[ s = \frac{(u_{out} - 2c_1 - 2c_2 + 6p_2 - a_{slx} - a_{sly})}{n_s} \]
a) Arranging studs

When $u_{out}$ is calculated, the perimeter can be drawn around the loaded area and the zone inside can be reinforced with shear studs. Care should be taken to ensure the shear reinforcement detailing rules described in section 9 are implemented.

The number of rails must be assumed in order to locate $u_{out}$. The centre line of each rail must be in line with centre point of the loaded area. $p_2$ is half of the loaded area diameter. The $u_{out}$ perimeter for circular loaded area is created with segments of lines offset by 1.5d from the last perimeter of the shear studs. Segments perpendicular to the radius of the loaded area are called ‘s’. The length of these segments is equal for the circular loaded area. The angle between the rail ($\alpha$) equates to:

$$\alpha = \frac{90}{ns}$$

Where:

$ns$ – the total number of ‘s’. The number of ‘s’ is defined by the 90° angle of each quarter being split into equal segments by the placing rails.

For equations to calculate ‘s’ in different conditions, see Section 11 (b).

In order to locate the $u_{out}$ perimeter, ‘p’ must be calculated:

$$p = s / (2 \sin(\alpha/2))$$

Therefore ‘p’ – the distance from the loaded area face to last stud equates to:

$$p_1 = p - p_2 - 1.5d / (\cos(\alpha/2))$$

When $p_1$ is worked out, the number of studs ($n_i$) on the rail can easily be calculated using the following formula:

$$n_i > \frac{(p_1 - 0.5d)}{0.75d}$$

The outcome should be rounded up to the nearest integer value.

The next step is to check the maximum tangential spacing between the studs on the rails in the following two cases:

- inside basic control perimeter check if $s_3 \leq 1.5d$
- outside basic control perimeter check if $s_{last} \leq 2d$

Where:

$s_3$ – the maximum tangential spacing inside the basic control perimeter (usually between the 3rd studs)

$s_{last}$ – the maximum tangential spacing between the studs outside the basic control perimeter

To check the tangential spacing, the distance from the centre point of the loaded area to the last stud inside the basic control perimeter (usually third) and the distance from the centre point of the loaded area to the last stud outside the basic control perimeter must be calculated.

$$r_3 = p_2 + \text{the distance to first stud} + 2 \cdot \text{studs spacing}$$

$$r_{last} = p_2 + \text{the distance to first stud} + (\text{number of studs on rail} - 1) \cdot \text{studs spacing}$$

Where:

$r_3$ – the distance to the 3rd stud

$r_{last}$ – the distance to the last stud

The spacing for both cases (inside and outside the basic control perimeter) should be calculated using the following formula:

$$s_t = \sqrt{(r + r \cos(\alpha))^2 + (r \sin(\alpha))^2}$$

When tangential spacing exceeds the maximum value, the number of rails should be increased or intermediate spacer rails should be used.
11. Circular Column – Circular Pattern

- Circular Pattern
- $0.3d - 0.5d$
- max $0.75d$
- $1.5d / (\cos(\alpha/2))$
- $p$
- $p_1$
- $p_2$
- max $0.75d$
- max $0.75d$
- max $0.75d$
- $0.3d - 0.5d$
- Rail A
- $u_1$
- $\alpha$
- $s = \text{max } 2d$
- $s = \text{max } 1.5d$
- $s_{\text{last}} = \text{max } 1.5d$
- Rail A
- $u_{\text{out}}$
- $1.5d$
- $c$
- $2d$
b) Calculating ‘s’ in different conditions

**Internal column**
\[ s = \frac{u_{out}}{n_s} \]

**Edge column**
\[ s = \frac{u_{out} - 2aslx}{n_s} \]

**External corner column**
\[ s = \frac{u_{out} - aslx - asly}{n_s} \]

**Internal corner column**
\[ s = \frac{u_{out} - aslx - asly}{n_s} \]
The design to EC2 using the cruciform pattern looks similar to the circular pattern design. The whole design methodology described in sections 3 to 9 is valid for cruciform pattern designs.

a) Arranging rails and studs

The key difference appears in the rails arrangement. As the cruciform pattern has no diagonal rails, the procedure of locating $u_{out}\ef$ looks slightly different. With the length of $u_{out}\ef$ we can locate the $u_{out}\ef$ perimeter. The minimum distance between the $u_{out}\ef$ perimeter and the centre point of the loaded area can be calculated using the following expression:

$$
p_{c\min} = u_{out}\ef / 8 - d \cdot (3\pi / 8 - 0.5 - \sqrt{2})
$$

The number of studs is given by the formula below (the value should be rounded up to the nearest integer):

$$
n_{tc} = (p_{c\min} - c_{1(2)} / 2 - \text{distance to first stud} - 1.5d) / 0.75d + 1
$$

The minimum distance ($F$) between the first and last rail on each side of the loaded area may be calculated using the following expression:

$$
F = u_{out}\ef / 4 - 0.75 \cdot \pi \cdot d - 2d
$$

The Distance $F$ should be infilled with rails taking into account the 1.5d spacing rule between rails. The drawing below illustrates the points above.

When all the rails are set, the final step is to calculate the required area of stud. The equations are identical to those for circular pattern designs (see section 8b for details).

Please note that only the rails with the first stud at 0.3$d$ to 0.5$d$ from the loaded area face and the last stud at 1.5$d$ from the $u_{out}\ef$ perimeter (shown in green above) can be taken to account when calculating the area of the stud (i.e. in the drawing above only 8 rails can be taken to account).
In cases where the loaded area is situated near a hole, "if the shortest distance between the perimeter of the loaded area and the edge of the hole does not exceed 6d, that part of the control perimeter contained between two tangents drawn to the outline of the hole from the centre of the loaded area is considered to be ineffective".  

(6.4.2 (3))

Because part of the $u_{out}$ control perimeter becomes ineffective ($u_{out \text{ Ineffective}}$), the additional control perimeter ($u_{out 2}$) must be found, the effective part of which will be equal to $u_{out}$ so:

$$u_{out 2} = u_{out \text{ Effective}} + u_{out \text{ Ineffective}}$$

In order to locate the $u_{out 2}$ control perimeter we have to assume that the control perimeter exists between the tangent lines as well. Therefore:

$$u_{out 2} = u_{out} / (1-\delta/360)$$

The part of the new control perimeter which is located outside the 'dead zone' ($u_{out 2}$) should equal the required $u_{out}$ control perimeter. The studs within the "dead zone" cannot be used in calculations which might cause an increase in the stud diameter required.

Rails around the "dead zone" can be cut or moved. Alternatively, additional rails might be placed to suit the hole and to comply with the spacing rules.

The drawings below show how to draw tangent lines. Please note that parts of the perimeters ($u_{0 \text{ red}}, u_{1 \text{ red}}, u_{out \text{ Ineffective}}$) located within the "dead zone" are ineffective and should be subtracted from the total length of the perimeter.

**Case 1 - hole dim $l_1 \leq l_2$**
Case 2 - hole dim $l_1 > l_2$

These studs cannot be used in calculations.
a) Circular Pattern - square column (internal condition)

Data
Slab depth $h = 325$ mm  
Column dimensions: $c_1 = 350$ mm, $c_2 = 350$ mm  
Load $V_{Ed} = 1070$ kN  
Cover = 25 mm (top and bottom)  
Reinforcement $T1$ & $T2 = H16 \ @ \ 175c/c$  
Compressive strength of concrete $f_{ck} = 30$MPa

Effective depth of the slab

dx = $325 - 25 - 16/2 = 292$ mm  
dy = $325 - 25 - 16 - 16/2 = 276$ mm  
d = $(dx + dy)/2 = (292 + 276)/2 = 284$ mm

Punching shear at the loaded area face

Eccentricity factor $\beta$ for internal column = 1.15  
$u_0 = 2\times c_1 + 2\times c_2 = 2\times 350 + 2\times 350 = 1400$ mm

$V_{Ed0} = \beta V_{Ed} / (u_0 \ d) = 1.15\times 1070\times 1000 / (1400\times 284) = 3.09$ MPa

$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c = 1\times 30 / 1.5 = 20$ MPa

$v_{Rd,\text{max}} = 0.3 f_{cd} (1 - (f_{ck} / 250)) = 0.3\times 20 (1 - (30 / 250)) = 5.28$ MPa

Check if $V_{Ed0} \leq v_{Rd,\text{max}}$  
$3.09$ MPa $< 5.28$ MPa  
Accepted.

Punching shear at the basic control perimeter without reinforcement

$u_1 = 2(c_1 + c_2) + 4\pi \ d = 2\times (350+350) + 4\times \pi \times 284 = 4968.8$ mm

$C_{Rd,\text{c}} = 0.18 / \gamma_c = 0.18 / 1.5 = 0.12$

$k = 1+ \sqrt{(200 / d)} = 1 + \sqrt{(200 / 284)} = 1.839 < 2$

$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2} = 0.035\times (1.839) ^{3/2} \times (30)^{1/2} = 0.478$ MPa

$V_{Ed1} = \beta V_{Ed} / (u_1 \ d) = 1.15\times 1070\times 1000 / (4968.8\times 284) = 0.872$ MPa

Consider reinforcement over $350 + 6\times 284 = 2.054$ m width in both directions from centre of column. For $\rho_l$ use $b = 1000$ mm.

Using $H16 \ @ \ 175$c/c in both directions = $1148.9$ mm$^2$/m  
$\rho_l = \sqrt((A_{tx} / (b \ d_x))\times A_{ty} / (b \ d_y)) = \sqrt((1148.9 / (1000-292))\times (1148.9 / (1000-276)) = 0.00405 < 0.02$

$v_{Rd,\text{c}} = C_{Rd,\text{c}} k (100 \rho_l f_{ck})^{1/3} = 0.12 \times 1.839 (100\times 0.00405\times 30)^{1/3} = 0.507$ MPa

Check if $v_{Rd,\text{c}} > v_{min}$  
$0.507$ MPa $> 0.478$ MPa

Check if $V_{Ed1} < v_{Rd,\text{c}}$  
$0.872$ MPa $< 0.507$ MPa  
Shear reinforcement required

Punching shear at the basic control perimeter with reinforcement

$\rho_{out,\text{req}} = \beta V_{Ed} / (V_{Rd,\text{c}} d) = 1.15\times 1070\times 1000 / (0.507\times 284) = 8541.5$ mm  
$350/2 = 175$ mm therefore: position rail central about the loaded area face in each direction.
therefore $p_2 = 175 \text{ mm}$

Try 12 rails.

$\alpha = \frac{360^\circ}{12} = 30^\circ$

$s = \frac{u_{\text{out}}}{12} = \frac{8541.5}{12} = 711.8 \text{ mm}$

$p = \frac{s}{(2 \cdot \sin(\alpha/2))} = \frac{711.8}{(2 \cdot \sin(30^\circ/2))} = 759.1 \text{ mm}$

$p_1 = p - p_2 - 1.5 \cdot \frac{d}{(\cos(\alpha/2))} = 1375.1 - 175 - 1.5 \cdot \frac{284}{(\cos(30^\circ/2))} = 759.1 \text{ mm}$

$0.75d = 0.75 \cdot 284 = 213 \text{ mm}$

$0.5d = 0.5 \cdot 284 = 142 \text{ mm}$

\[ \text{stud spacing} = 210 \text{ mm}, \text{ distance to first stud} = 140 \text{ mm} \]

**Stud spacing check**

Distance to 3rd stud $r_3 = 175/ \cos 30^\circ + 140 + 2 \cdot 210 = 762.1 \text{ mm}$

Distance to last stud $r_{\text{last}} = 175/ \cos 30^\circ + 140 + 3 \cdot 210 = 972.1 \text{ mm}$

$s_{3,4} = \sqrt{(r_3 - r_3 \cos(\alpha))^2 + (r_3 \sin(\alpha))^2} = \sqrt{((762.1 - 762.1 \cdot \cos(30^\circ))^2 + (762.1 \cdot \sin(30^\circ))^2)} = 394.5 \text{ mm} < 1.5d = 426 \text{ mm}$

$s_{\text{last}} = \sqrt{(r_{\text{last}} - r_{\text{last}} \cos(\alpha))^2 + (r_{\text{last}} \sin(\alpha))^2} = \sqrt{((972.1 - 972.1 \cdot \cos(30^\circ))^2 + (972.1 \cdot \sin(30^\circ))^2)} = 503.2 \text{ mm} < 2d = 568 \text{ mm}$

**Area of shear reinforcement**

$f_{\text{wrd,ef}} = 250 + 0.25d = 250 + 0.25 \cdot 284 = 321 \text{ N/mm}^2$

$f_{\text{wrd}} = \frac{f_y}{1.15} = 500 / 1.15 = 434.8 \text{ N/mm}^2 > 321$

$s_t = 503 \text{ mm}, s_s = 210 \text{ mm}, \text{ rail no. taken as 12}$

$A_{\text{sw,min}} = 0.08 s_t s_s \sqrt{f_{\text{ck}}} / (1.5 f_y) = 0.08 \cdot 503 \cdot 210 \sqrt{30} / (1.5 \cdot 500) = 61.7 \text{ mm}^2$

$A_{\text{sw1}} = (V_{\text{Ed,1}} - 0.75 V_{Rd,c}) / u_1 s_t / (1.5 f_{\text{wrd,ef}} \text{ rail no.})$

$A_{\text{sw1}} = (0.872 - 0.75 \cdot 0.507) \cdot 4968.8 \cdot 210 / (1.5 \cdot 321.0 \cdot 12) = 88.8 \text{ mm}^2$

\[ \text{stud dia} = 12 \text{ mm} (A = 113.1 \text{ mm}^2) \]

Provide

12 No 12-4-275-910 (1357.2 mm$^2$).

Spacing:

140/210/210/210/140

48 Studs total
a) Circular Pattern - circular column with hole (edge condition)

Data
Slab depth \( h = 300 \text{ mm} \)
Slab edge offset \( a_{slx} = 900 \text{ mm} \)
Column dia \( c = 300 \text{ mm} \)
Load \( V_{Ed} = 610 \text{ kN} \)
Cover = 30 mm (top and bottom)
Reinforcement \( T1 = H16 @ 150c/c \)
Reinforcement \( T2 = H16 @ 150c/c \)
Compressive strength of concrete \( f_{ck} = 30\text{MPa} \)
Yield strength of reinforcement \( f_y = 500\text{MPa} \)
Hole data - as per drawing

Effective depth of the slab
\[
\begin{align*}
\text{dx} &= 300 - 30 - 16/2 = 262 \text{ mm} \\
\text{dy} &= 300 - 30 - 16 - 16/2 = 246 \text{ mm} \\
d &= (\text{dx} + \text{dy})/2 = (262 + 246)/2 = 254 \text{ mm}
\end{align*}
\]

Punching shear at the loaded area face
\[
\begin{align*}
\text{aslx} &= 900 < \pi/4 \cdot 4 \cdot (c + 4 \cdot d) = \pi/4 \cdot (300 + 4 \cdot 254) = 1033.6 \\
\text{Therefore by interpolation eccentricity factor } \beta &= 1.188 \\
\text{Angle } \delta &= 2 \cdot \arctan(((100)/(500))) = 22.6^\circ \\
\text{u0}_{\text{red}} &= 22.6/360 \cdot \pi \cdot c = 59.2 \text{ mm} \\
\text{u0} &= 0.25 \cdot \pi \cdot c + B_N + B_S + B_W - \text{u0}_{\text{red}} = 883.3 \text{ mm} \\
V_{Ed,0} &= \beta \cdot V_{Ed} / (\text{u0} \cdot d) = 3.23 \text{ MPa} \\
f_{cd} &= \alpha_{cc} \cdot f_{ck} / \gamma_c = 20 \text{ MPa} \\
V_{Rd,\text{max}} &= 0.3 \cdot f_{cd} (1 - (f_{ck} / 250)) = 5.28 \text{ MPa}
\end{align*}
\]

Check if \( V_{Ed,0} \leq V_{Rd,\text{max}} \) \( \leq 3.23 \text{ MPa} < 5.28 \text{ MPa} \) \( \checkmark \) Accepted.

Punching shear at the basic control perimeter without reinforcement
\[
\begin{align*}
\text{u}_{1A} &= (c + 4d) \cdot \pi \cdot (u_{\text{1A,red}} = (300 + 4 \cdot 254) \cdot \pi - 259.8) = 3874.6 \text{ mm} \\
\text{u}_{1B} &= (c + 4d) \cdot \pi/2 + 2 \cdot a_{slx} - u_{\text{1B,red}} = (300 + 4 \cdot 254) \cdot \pi/2 + 900 - 261.5 = 3605.7 \text{ mm} \\
\text{C}_{Rd,c} &= 0.18 / \gamma_c = 0.18 / 1.5 = 0.12 \\
\text{k} &= 1 + \sqrt{(200 / d)} = 1.887 < 2 \\
\text{v}_{\text{min}} &= 0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2} = 0.035 \cdot (1.887)^{3/2} \cdot (30)^{1/2} = 0.497 \text{ MPa} \\
V_{Ed,1} &= \beta \cdot V_{Ed} / (u_{1d}) = 0.791 \text{ MPa}
\end{align*}
\]

Consider reinforcement over \( 300 + 6 \cdot 254 = 1824 \text{ mm} \) width in both directions from centre of the loaded area. For \( \rho_l \) use \( b = 1000 \text{mm} \).
Using H16 @ 150c/c in both directions = 1340.4 mm²/m  
Using H16 @ 150c/c in both directions = 1340.4 mm²/m

\[ \rho_l = \sqrt{\left( \frac{A_{sx}}{b \cdot dx} \right) \cdot \left( \frac{A_{sy}}{b \cdot dy} \right)} = \sqrt{1340.4 / (1000 \cdot 262) \cdot 1340.4 / (1000 \cdot 246)} = 0.00528 < 0.02 \]

\[ v_{Rd.c} = C_{Rd.c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{\text{ok}})^{1/3} = 0.12 \cdot 1.887 \cdot (100 \cdot 0.00528 \cdot 30)^{1/3} = 0.569 \text{ MPa} \]

Check if \( v_{Rd.c} > v_{\text{min}} \) 0.569 MPa > 0.497 MPa
Check if \( v_{Ed.1} < v_{Rd.c} \) 0.791 MPa < 0.569 MPa \( \Box \) Shear reinforcement required
Punching shear at the basic control perimeter with reinforcement

\[ u_{\text{out req.}} = \beta \frac{V_{Ed}}{(v_{Rd,c} \cdot d)} = 1.188 \cdot 610 \cdot 1000 / (0.569 \cdot 254) = 5015 \text{ mm} \]

Try 2 mid spur in quarter & no. of s = (2+1)\cdot2 = 6

\[ s = \frac{(u_{\text{out req.}} - 2\cdot a_{\text{slx}})}{\text{no. of s}} = (5015 - 2\cdot 900) / 6 = 535.8 \text{ mm} \]

\[ \alpha = 90 / (\text{mid spurs in quarter} + 1) = 90 / 3 = 30^\circ \]

\[ p = s / (2 \cdot \sin (\alpha/2)) = 535.8 / (2 \cdot \sin (30/2)) = 1035.1 \text{ mm} \]

Angle \[ \delta = 22.6^\circ \]

\[ \gamma = 262.0^\circ \]

\[ u_{\text{out2 req.}} = \frac{u_{\text{out req.}}}{1 - \frac{\delta}{\gamma}} = \frac{5015}{1 - 22.6^\circ / 262.0^\circ} = 5489.1 \text{ mm} \]

\[ s = \frac{(u_{\text{out2 req.}} - 2\cdot a_{\text{slx}})}{\text{no. of s}} = (5489.1 - 2\cdot 900) / 6 = 614.8 \text{ mm} \]

\[ p = s / (2 \cdot \sin (\alpha/2)) = 614.8 / (2 \cdot \sin (30/2)) = 1187.8 \text{ mm} \]

\[ p_2 = c/2 = 300/2 = 150 \text{ mm} \]

\[ p_1 = p - p_2 - 1.5d / (\cos (\alpha/2)) = 1187.8 - 150 - 1.5 \cdot 254 / (\cos (30/2)) = 643.4 \text{ mm} \]

\[ 0.75d = 0.75 \cdot 254 = 190.5 \text{ mm} \]

\[ 518.4 / 190.5 = 2.72 \]

\[ 518.4 / 3 = 172.8 \]

\[ \text{stud spacing} = 175 \text{ mm}, 4 \text{ studs on a rail.} \]

Stud spacing check

Rail A - distance to last stud \[ r_{3,A} = 150 + 125 + 2\cdot175 = 625 \text{ mm} \]

\[ s_{r3} = 2\cdot r_{3,A} \cdot \sin(\alpha/2) = 2\cdot 625 \cdot \sin(30/2) = 323.5 \text{ mm} < 1.5d = 381 \text{ mm} \]

Rail A - distance to last stud \[ r_{\text{last A}} = 150 + 125 + 3\cdot175 = 800 \text{ mm} \]

\[ s_{r\text{last}} = 2\cdot r_{\text{last A}} \cdot \sin(\alpha/2) = 2\cdot 800 \cdot \sin(30/2) = 414.1 \text{ mm} < 2d = 508 \text{ mm} \]

Area of shear reinforcement

\[ f_{yw}\cdot ef = 250 + 0.25d = 250 + 0.25\cdot 254 = 313.5 \text{ N/mm}^2 \]

\[ f_{yw} = (f_y / 1.15) = 500 / 1.15 = 434.8 \text{ N/mm}^2 > 313.5 \]

\[ s_r = 414.1 \text{ mm}, s = 175 \text{ mm}, \text{ rail no. taken as 7 (for rail within } u_1 \text{ perimeter)} \]

\[ A_{sw,\text{min}} = 0.08 l_s \cdot s_r \cdot f_{ck} / (1.5 \cdot f_y) = 0.08 \cdot 414.1 \cdot 175 \cdot \sqrt{30} / (1.5 \cdot 500) = 42.3 \text{ mm}^2 \]

\[ A_{sw1} = (v_{Ed,1} - 0.75 v_{Rd,c}) \cdot u_1 \cdot s_r / (1.5 \cdot f_{yw,ef} \cdot \text{rail no.}) \]

\[ A_{sw1} = (0.791 - 0.75 \cdot 0.569 \cdot 3605.7 \cdot 175) / (1.5 \cdot 313.5 \cdot 7) = 69.9 \text{ mm}^2 \]

\[ \text{stud dia} = 10 \text{ mm} \quad (A = 78.5 \text{ mm}^2) \]
Provide:
13 No 10-4-240-775. Spacing: 125/175/175/175/125
2 No 10-3-240-600. Spacing: 125/175/175/125
58 Studs in total.
a) Circular Pattern - square column

Data
Slab depth $h = 400$ mm
Column dimensions: $c_1=300$ mm, $c_2=300$ mm
Load $V_{Ed} = 320$ kN
Cover = 30mm (top and bottom)
Reinforcement T1 & T2 = H16 @ 150c/c
Compressive strength of concrete $f_{ck} = 30$MPa
Distance to slab edge $a_{slx} = 0$ mm, $a_{sly} = 0$ mm

Effective depth of the slab

d$x = 400 - 30 - 16/2 = 362$ mm  
d$y = 400 - 30 - 16 - 16/2 = 346$ mm  
d = $(d_x + d_y)/2 = (362 + 346)/2 = 354$ mm

Punching shear at the loaded area face

$B_S = \min. (c_1, c_1 + a_{slx}, 1.5d) = \min. (300, 300, 531) = 300$ mm  
$B_E = \min. (c_2, c_2 + a_{sly}, 1.5d) = \min. (300, 300, 531) = 300$ mm  
$u_0 = B_S + B_E = 300 + 300 = 600$mm  
$v_{Ed,0} = \beta \frac{V_{Ed}}{(u_0 d)} = 1.5 \cdot \frac{320 \cdot 1000}{(600 \cdot 354)} = 2.26$ MPa  
$f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma_c} = 1 \text{•} \frac{30}{1.5} = 20$ MPa  
$v_{Rd,max} = 0.3 f_{cd} (1 - (f_{ck} / 250)) = 0.3 \cdot 20 (1 - (30 / 250)) = 5.28$ MPa

Check if $v_{Ed,0} \leq v_{Rd,max} \implies 2.26 \text{ MPa} < 5.28 \text{ MPa} \implies \text{Accepted.}$

Punching shear at the basic control perimeter without reinforcement

$u_1 = c_1 + c_2 + \pi \cdot d = 300 + 300 + \pi \cdot 254 = 1712.1$ mm  
$C_{Rd,c} = 0.18 / \gamma_c = 0.18 / 1.5 = 0.12$  
k = $1 + \sqrt{(200 / d)} = 1 + \sqrt{(200 / 354)} = 1.752 < 2$  
$v_{min} = 0.035 k^{3/2} f_{dk}^{1/2} = 0.035 \cdot (1.752)^{3/2} (\frac{30}{354})^{1/2} = 0.444$ MPa  
$v_{Ed,1} = \beta \frac{V_{Ed}}{(u_1 d)} = 1.5 \cdot \frac{320 \cdot 1000}{(1712.1 \cdot 354)} = 0.792$ MPa

Consider reinforcement over $300 + 3 \cdot 354 = 1362$ mm width in both directions from centre of the loaded area. For $\rho_l$ use $b = 1000$mm.

Using H16 @ 150c/c in both directions = 1340.4 mm$^2$/m  
$\rho_l = \sqrt{(A_{as} / (b d_y) A_{sy} / (b d_y)) = \sqrt{(1340.4 / 362) \cdot 362 / 346}} / 1000 = 0.00379 < 0.02$  
$v_{Rd,c} = C_{Rd,c} k (100 \rho_l f_{dk})^{1/3} = 0.12 \cdot 1.935 (100 - 0.00586 - 30)^{1/3} = 0.604$ MPa

Check if $v_{Rd,c} > v_{min} \implies 0.473$ MPa $> 0.444$ MPa
Check if $v_{Ed,1} < v_{Rd,c} \implies 0.792$ MPa $< 0.473$ MPa $\implies \text{Shear reinforcement required.}$
Punching shear at the basic control perimeter with reinforcement

\[ u_{\text{out req}} = \beta \frac{V_{Ed}}{(\nu_{Rd,c} d)} = 2869.4 \text{ mm} \]

Therefore \( p_2 = 150 \text{ mm} \)

Try 4 rails.

\[ \alpha = \frac{90^\circ}{(4-1)} = 30^\circ \]

\[ s = \frac{(u_{\text{out}} - 2p_2)}{(4-1)} = \frac{(2869.4 - 2 \cdot 150)}{3} = 856.5 \text{ mm} \]

\[ p = s/(2\sin(\alpha/2)) = 856.5 / (2\sin(30^\circ/2)) = 1654.6 \text{ mm} \]

\[ p_1 = p - p_2 - 1.5d / (\cos(\alpha/2)) = 1654.6 - 150 - 1.5 \cdot 354 / (\cos(30^\circ/2)) = 954.8 \text{ mm} \]

\[ 0.75d = 0.75 \cdot 354 = 265.5 \text{ mm} \]

\[ 0.5d = 0.5 \cdot 354 = 177 \text{ mm} \]

\[ \text{stud spacing} = 260 \text{ mm}, \text{distance to first stud} = 175 \text{ mm} \]

Stud spacing check

Distance to 3rd stud \( r_3 = 150 / \cos(30^\circ) + 175 + 2 \cdot 260 = 868.2 \text{ mm} \)

Distance to last stud \( r_{\text{last}} = 150 / \cos(30^\circ) + 175 + 3 \cdot 260 = 1128.2 \text{ mm} \)

\[ s_{r3} = \sqrt((r_3 - r_3 \cos(\alpha))^2 + (r_3 \sin(\alpha))^2) = 449.4 \text{ mm} < 1.5d = 531 \text{ mm} \]

\[ s_{r\text{last}} = \sqrt((r_{\text{last}} - r_{\text{last}} \cos(\alpha))^2 + (r_{\text{last}} \sin(\alpha))^2) = 584.0 \text{ mm} < 2d = 708 \text{ mm} \]

Area of shear reinforcement

\[ f_{yw,\text{eff}} = 250 + 0.25 \cdot d = 250 + 0.25 \cdot 354 = 338.5 \text{ N/mm}^2 \]

\[ f_{yw,\text{eff}} = (f_y / 1.15) = 500 / 1.15 = 434.8 \text{ N/mm}^2 > 338.5 \]

\[ s_t = 584.0 \text{ mm}, s_r = 260 \text{ mm}, \text{rail no. taken as 4} \]

\[ A_{sw,\text{min}} = 0.08 \cdot s_t \cdot s_r \cdot \frac{\sqrt{f_y}}{(1.5f_y)} = 0.08 \cdot 584.0 \cdot 260 \cdot \sqrt{30} / (1.5 \cdot 500) = 88.7 \text{ mm}^2 \]

\[ A_{sw,1} = (V_{Ed,1} / 0.75 \cdot V_{Rd,c}) \cdot u_1 \cdot s_r / (1.5 \cdot f_{yw,\text{eff}} \cdot \text{rail no.}) \]

\[ A_{sw,1} = (0.792 - 0.75 \cdot 0.473) \cdot 1712.1 \cdot 260 / (1.5 \cdot 338.5 \cdot 4) = 95.9 \text{ mm}^2 \]

\[ \text{stud dia} = 12 \text{ mm} (A = 113.1 \text{ mm}^2) \]

Provide 4 No 12-4-340-1130

Spacing: 175/260/260/260/175 (452.4 mm²) 16 Studs total

Rail layout
a) Circular Pattern - square column

Data
Slab depth \( h = 350 \text{ mm} \)
Column dimensions: \( c_1=400 \text{ mm}, c_2=400 \text{ mm} \)
Load \( V_{Ed} = 1100 \text{ kN} \)
Cover = 25 mm (top and bottom)
Reinforcement T1 & T2 = H20 @ 175c/c
Compressive strength of concrete \( f_{ck} = 40\text{MPa} \)
Distance to slab edge \( a_{slx} = 0 \text{ mm}, a_{sly} = 0 \text{ mm} \)

Effective depth of the slab
\[
\begin{align*}
d_x &= 350 - 25 - 20/2 = 315 \text{ mm} \\
d_y &= 350 - 25 - 20 - 20/2 = 295 \text{ mm} \\
d &= \frac{(d_x + d_y)}{2} = \frac{(315 + 295)}{2} = 305 \text{ mm}
\end{align*}
\]

Punching shear at the loaded area face
\[
\begin{align*}
a_{slx} &= 0, a_{sly} = 0 \square \text{ eccentricity factor } \beta = 1.275 \\
B_N &= \min. (c_1, c_1 + a_{slx}, 1.5d) = \min. (400, 400, 457.5) = 400 \text{ mm} \\
B_W &= \min. (c_2, c_2 + a_{sly}, 1.5d) = \min. (400, 400, 457.5) = 400 \text{ mm} \\
u_0 &= c_1 + c_2 + B_N + B_W = 400 + 400 + 400 + 400 = 1600\text{mm} \\
V_{Ed,0} &= \beta \frac{V_{Ed}}{(u_0 d)} = 1.275 \times \frac{1100 \times 1000}{1600 \times 305} = 2.87 \text{ MPa} \\
f_{cd} &= \alpha_{cc} \frac{f_{ck}}{\gamma_c} = 1 \times \frac{40}{1.5} = 26.67 \text{ MPa} \\
v_{Rd,max} &= 0.3 f_{cd} \left(1 - \frac{(f_{ck}}{250}\right)) = 0.3 \times 26.67 \times (1 - \frac{40}{250}) = 6.72 \text{ MPa}
\end{align*}
\]
Check if \( V_{Ed,0} \leq v_{Rd,max} \square 2.87 \text{ MPa} < 6.72 \text{ MPa} \square \text{ Accepted.}

Punching shear at the basic control perimeter without reinforcement
\[
\begin{align*}
u_1 &= 2c_1 + 2c_2 + 3\pi d = 2 \times 400 + 2 \times 400 + 3 \times \pi \times 305 = 4474.6 \text{ mm} \\
C_{Rd,c} &= 0.18 \times \frac{1}{\gamma_c} = 0.18/1.5 = 0.12 \\
k &= 1 + \sqrt{(200 / d)} = 1 + \sqrt{(200/305)} = 1.81 < 2 \\
\nu_{min} &= 0.035 k^{3/2} f_{ck}^{1/2} = 0.035 \times (1.81)^{3/2} \times (40)^{1/2} = 0.539 \text{ MPa} \\
v_{Ed,1} &= \beta \frac{V_{Ed}}{(u_1 d)} = 1.275 \times \frac{1100 \times 1000}{4474.6 \times 305} = 1.028 \text{ MPa}
\end{align*}
\]
Consider reinforcement over \( 400 + 3 \times 305 = 1315 \text{ mm} \) width in both directions from centre of the loaded area. For \( \rho_l \) use \( b = 1000\text{mm} \).
Using H20 @ 175c/c in both directions = 1795.2 \text{ mm}^2/m
\[
\begin{align*}
\rho_l &= \sqrt{\frac{(A_{ax} / (b d_x) - A_{sy} / (b d_y))}{(1795.2 / 315\times1795.2/295) / 1000}} = 0.00589 < 0.02 \\
v_{Rd,c} &= C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} = 0.12 \times 1.810 (100 \times 0.00589 \times 40)^{1/3} = 0.623 \text{ MPa}
\end{align*}
\]
Check if \( v_{Rd,c} \geq v_{min} \square 0.623 \text{ MPa} > 0.539 \text{ MPa} \)
Check if \( v_{Ed,1} < v_{Rd,c} \square 1.028 \text{ MPa} < 0.623 \text{ MPa} \square \text{ Shear reinforcement required} \)
**Punching shear at the basic control perimeter with reinforcement**

\[ \text{u}_{\text{out required}} = \beta \frac{V_{Ed}}{V_{Rd,c}} \text{d} = 1.275 \times 1100 \times 1000 / (0.623 \times 305) \]

\[ = 7386.3 \text{ mm} \]

\[ 400 / 2 = 200 \text{ mm} \]

Therefore \( p_2 = 200 \text{ mm} \)

Try 4 rails in quarter.

\[ \alpha = \frac{90^\circ}{4-1} = 30^\circ \]

\[ s = \frac{(u_{\text{out}} - 2p_2)}{9} = \frac{(7386.3 - 2 \times 150)}{9} \]

\[ = 776.3 \text{ mm} \]

\[ p = \frac{s}{2 \times \sin(\alpha / 2)} = \frac{776.3}{2 \times \sin(30^\circ / 2)} = 1499.6 \text{ mm} \]

\[ p_1 = p - p_2 - \frac{1.5 \text{d}}{\cos(\alpha / 2)} = 1499.6 - 200 - \frac{1.5 \times 305}{\cos(30^\circ / 2)} = 826.0 \text{ mm} \]

\[ 0.75 \text{d} = 0.75 \times 305 = 228.75 \text{ mm} \]

\[ 0.5 \text{d} = 0.5 \times 305 = 152.5 \text{ mm} \]

\[ \text{stud spacing} = 230 \text{ mm}, \text{distance to first stud} = 150 \text{ mm} \]

**Stud spacing check**

Distance to 3rd stud: \( r_3 = 200 / \cos 30^\circ + 150 + 2 \times 230 \)

\[ = 840.9 \text{ mm} \]

Distance to last stud: \( r_{\text{last}} = 200 / \cos 30^\circ + 150 + 3 \times 230 \)

\[ = 1070.9 \text{ mm} \]

\[ s_{r_3} = \sqrt{(r_3 - r_{r_3} \cos(\alpha))^2 + (r_3 \sin(\alpha))^2} = \sqrt{(840.9 - 840.9 \times \cos(30^\circ))^2 + (840.9 \times \sin(30^\circ))^2} = 435.3 \text{ mm} < 1.5 \text{d} = 457.5 \text{ mm} \]

\[ s_{r_{\text{last}}} = \sqrt{(r_{r_{\text{last}}} - r_{r_{\text{last}}} \cos(\alpha))^2 + (r_{r_{\text{last}}} \sin(\alpha))^2} = \sqrt{(1070.9 - 1070.9 \times \cos(30^\circ))^2 + (1070.9 \times \sin(30^\circ))^2} = 554.4 \text{ mm} < 2 \text{d} = 610 \text{ mm} \]

**Area of shear reinforcement**

\[ f_{\text{w,d,ef}} = 250 + 0.25 \text{d} = 250 + 0.25 \times 305 \]

\[ = 326.3 \text{ N/mm}^2 \]

\[ f_{\text{w,d}} = \frac{f_y}{1.15} \]

\[ = 500 / 1.15 \]

\[ = 434.8 \text{ N/mm}^2 > 326.3 \]

\[ s_i = 554.4 \text{ mm}, s_r = 230 \text{ mm}, \text{rail no. taken as 12} \]

\[ A_{\text{sw,min}} = 0.08 s_i s_r \sqrt{f_{\text{ck}} / (1.5 f_{\psi_k})} = 0.08 \times 554.4 \times 230 \sqrt{40} / (1.5 \times 500) = 86.0 \text{ mm}^2 \]

\[ A_{\text{sw,1}} = (0.75 \text{d} / \text{F}_{\text{Rd,c}}) \text{u}_i \text{s}_i / (1.5 f_{\text{w,d,ef}} \text{rail no.}) \]

\[ = (1.028 - 0.75 \times 0.623) \times 4474.6 \times 230 / (1.5 \times 326.3 \times 12) = 98.3 \text{ mm}^2 \]

\[ \text{Provide 12 No 12-4-300-990} \]

**Spacing:** 150/230/230/230/150

\( (1357.2 \text{ mm}^2) 48 \text{ Studs total} \)

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**Rail Layout**

17. Example Calculation – Internal Corner Condition